## Problem 1

Consider a system that consists of two finite-dimensional linear subsystems described by the state-space equations:

|  |  |
| --- | --- |
|  | (1) |

The orders of the systems are  and  respectively. It is assumed that

|  |  |
| --- | --- |
|  | (2) |

Let the inner time delay between two subsystems be

|  |  |
| --- | --- |
|  | (3) |

where d is an integer, h is the sampling period, and  is a fraction of the sampling interval.

|  |  |
| --- | --- |
|  | (4) |

For time delay between the subsytems, equation (1) is often described as

|  |  |
| --- | --- |
|  | (5) |

whereis set as 0 for convenient calculations, and=.

By integrating the differential equations, we obtain the sampled representation as follows:

|  |  |
| --- | --- |
|  | (6) |

|  |  |
| --- | --- |
|  | (7) |

In the interval 

|  |  |
| --- | --- |
|  | (8) |

Thus,

|  |  |
| --- | --- |
|  | (9) |

where

|  |  |
| --- | --- |
|  | (10) |

|  |  |
| --- | --- |
|  | (11) |

The sampled version of (5) is thus

|  |  |
| --- | --- |
|  | (12) |

It is clear from the derivation that if there is a delay in the system, then  in (7) is replaced by . It is also easily seen that if , then (9) is changed to

|  |  |
| --- | --- |
|  | (13) |

While treating the case when the delay is not a multiple of the sampling period and without loss of generality, it can be assumed that .

By considering  and sampling the system as the same problem treated in Case 1, where there is a time delay before the system, equation (12) and equation (14) can be rewritten as

|  |  |
| --- | --- |
|  | (14) |

|  |  |
| --- | --- |
|  | (15) |

According to  and equation (16) from Case 2, where the time delay is after the system, we can express as follows:

|  |  |
| --- | --- |
|  | (16) |

|  |  |
| --- | --- |
|  | (17) |

From equation (12) and equation (17), we can conclude that the periodic sampling of a system that has time delay between the subsystems with the sampling interval h and with  gives the sampled data representation

|  |  |
| --- | --- |
|  | (18) |

where

|  |  |
| --- | --- |
|  | (19) |

and

|  |  |
| --- | --- |
|  | (20) |

## Problem 2-2

Calculate the values for F and H according to given values in equation (21).

|  |  |
| --- | --- |
|  | (21) |

|  |  |
| --- | --- |
|  | (22) |

|  |  |
| --- | --- |
|  | (23) |

From equation (21), equation (22), and equation (23), the discrete system can be described as follows:

|  |  |
| --- | --- |
|  | (24) |

## Problem 3

Substitute k+2 with k:

|  |  |
| --- | --- |
|  | (25) |

Then calculate the value of :

|  |  |
| --- | --- |
|  | (26) |

We then compute the output sequence:

|  |  |
| --- | --- |
|  | (27) |

Calculate the values of a and b.

|  |  |
| --- | --- |
|  | (28) |

We then get the result of the output sequence.

|  |  |
| --- | --- |
|  | (29) |

.

## Problem 2-4

According to the question, we calculate the values for F, , and using Matlab.

|  |  |
| --- | --- |
|  | (30) |

|  |  |
| --- | --- |
|  | (31) |

|  |  |
| --- | --- |
|  | (32) |

|  |  |
| --- | --- |
|  | (33) |

|  |  |
| --- | --- |
|  | (34) |

From equations above, we get the pulse-transfer operator 

References

[1: Wittenmark 1985]

Bjorn Wittenmark, “Sampling of a system with a time delay,” IEEE Transactions on Automatic Control, Vol. 30, No. 5, pp. 507-510, May 1985